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FRACTURE DIAGRAMS OF PRISMATIC SPECIMENS WITH A REFINED CONTACT STRESS DISTRIBUTION LAW

[1]Malich M., [2]Katan V., [1]Laikov D., [3]Kravchenko K.

[1]*Ukrainian State University of Science and Technology*
[2]*Oles Honchar Dnipro National University*
[3]*Dnipro University of Technology*

Abstract. The extraction and further processing of minerals are associated with the improvement of existing and development of new resource-saving technological solutions for underground and open-cycle operationsThe most costly technology for processing mineral raw materials is destruction itself, which accounts for about 20% of total electricity production and up to 50% of total capital and operating costs. To determine the stability of the rock mass in an ultimate state, a stress-strain diagram of rock is used. To analyse these diagrams, it is necessary to know the distribution of contact normal and tangential stresses. According to classical solutions to this problem, the stress distribution was determined using the methods of L. Prandtl and E.P. Unksov. However, these methods did not account for the occurrence of stresses perpendicular to the compression vector.

The article provides further development of the method for refining the distribution of contact stresses and constructing a “normal stress-longitudinal deformation” diagram and limit curves, taking into account contact friction under load. A comparative assessment of the proposed method with diagrams constructed using the classical method is carried out.. The proposed method allows determining the strength limit and residual strength of rock samples based on parameters that can be easily established experimentally in the laboratories of mining enterprises. The results can be used to monitor the condition of the rock mass and ensure effective rock destruction.

Therefore, an improved method for determining the distribution of contact normal and tangential stresses in prismatic samples of brittle, relatively homogeneous rocks under stress is proposed, which takes into account stresses perpendicular to the load vector and allows for a more accurate assessment of the stress-deformed state of the samples taking into account contact. It has been established that, especially at small angles of internal friction, the level of current stresses according to the proposed method is lower than that obtained using classical solutions.

The developed approach allows avoiding the influence of the scale effect and transferring the results of laboratory studies to real, undamaged and relatively homogeneous arrays. The results are of practical importance for assessing the stability of rock masses and improving the efficiency of rock destruction during the extraction and processing of solid minerals.

Keywords: contact, friction, diagram, rock, deformation, stress, destruction

1. Introduction

The territory of Ukraine has numerous deposits of ore and non-ore minerals, including coal and granite deposits in the mesomorphic and metamorphic rock mass of the Ukrainian Shield. These masses are widely used in the construction, coal, and metallurgical industries. The extraction and further processing are linked to the improvement of existing and development of new resource-saving technological solutions for underground and open-cycle operations. This is accompanied by the need to ensure the stability of the rock mass on the one hand, and to increase the efficiency of rock disintegration on the other hand. Ensuring rock mass stability is important for coal mining, especially with increasing mining depth, when rock pressure increases significantly, rock properties change, and geotechnical processes in the rock mass around the workings are significantly activated, accompanied by human and material losses. At the same time, the most costly technology for processing mineral raw materials is disintegration itself, to which billions of tons are subjected. These processes account for about 20% of total electricity production and up to 50% of the total capital and operating costs of mining enterprises. In this regard,

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there is a need for further research into the processes of rock destruction by improving the accuracy of assessing its bearing capacity under various types of force.

In the mining and processing industries, when assessing the mechanical properties of rocks, their strength is determined based on production drilling data obtained using various machines and tools. Tests are conducted under different technological conditions, which leads to generalisation and averaging of results, discrepancies between the actual strength of rocks and the values accepted at mines, enrichment plants, and quarries. The strength limit of rock is determined using stress–strain diagrams during uniaxial compression of samples on special equipment [1–5]. However, the machines are expensive, difficult to operate, require highly skilled maintenance, and are only available in specialised research institutes in Ukraine. Therefore, there is an obvious need for an analytical method for calculating the strength limit and residual strength of rock samples, which could be carried out by simple means directly at the mining site [6, 7].

Known works on mathematical modelling of sample destruction [8, 9] have not been developed into complete analytical methods for calculating the parameters of "normal stress – longitudinal deformation" diagrams beyond the strength limits. In [10], Coulomb's strength criterion was used to model failure, taking into account contact friction, but the rule of parity of tangential stresses in angular zones was not considered. In [11], the formation of a barrel shape due to the inhibition of transverse deformation by contact friction between the press plate and the sample, which gives grounds for applying the rule of parity of tangential stresses in corner zones during deformation and failure of the sample, is used to refine the distribution of contact normal and tangential stresses.

The work aims to develop an improved analytical method for determining the distribution of contact normal and tangential stresses in prismatic rock samples under compression, taking into account normal stresses that are neglected in classical approaches (e.g., Prandtl or E.P. Unksov). By improving the construction of the stress–strain diagram and taking into account the effects of contact friction, the proposed method increases the accuracy of fracture prediction and residual strength assessment. The results provide a practical tool for assessing the stability of rock masses and optimising the processes of destruction during the extraction and enrichment of minerals using experimentally measured parameters (shear strength, friction coefficients, elastic modulus).

2. Methods

In the theory of metal deformation under pressure, it is recommended to determine the distribution of contact stresses during the deformation of a metal strip of unlimited length by solving two differential equations of balance and an algebraic equation of plasticity (limit state) of metal.

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \qquad (1)$$

$$\frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{xy}}{\partial x} \qquad (2)$$

$$\sigma_x-\sigma_y=2k_n\sqrt{1-\frac{\tau_{xy}^2}{k_n^2}} \qquad (3)$$

where k_n – is the shear strength of the material (constant plasticity), Pa; τ_{xy} – is the tangential stress, Pa; σ_x – is the horizontal normal stress, Pa; σ_y – is the vertical normal stress, Pa.

The boundary conditions for system (1–3) are the condition of compressive loading of the strip and the absence of loading on the side. However, accounting contact friction creates insurmountable difficulties in the theory of metal processing by pressure when accurately integrating these three equations. As a result, when solving practical problems involving the determination of deforming forces, certain simplifications must be introduced. In [12], based on the theory of plastic deformation, a detailed analysis of the possibility of simplifying the equations was carried out and the limits of their applicability within the limits of practically acceptable accuracy were determined experimentally. To do this: a) the problem is reduced to a symmetrical problem or plane one; b) it is assumed that normal stresses depend only on one coordinate, in particular on *x*, and the dependence of tangential stresses on the coordinate *y* is assumed to be linear.

As a result, the differential equations are simplified. Their number is reduced to one, which contains simple derivatives instead of partial ones.

Consider the second equation (2) of the system. It is satisfied if σ_y does not depend on *y*. The contact tangential stresses τ_k are assumed to be equal to the limiting value of constant plasticity k_n. Then equation (3) is reduced to the expression $\frac{\partial \sigma_y}{\partial x}=\frac{\partial \sigma_y}{\partial y}$. It should be noted that the latter expression is recommended to be used under the condition $\tau_k \geq 0.7k_n$. Then the stresses at the tip of the crack are

$$\sigma_{y_i}=\sigma_{y_0}e^{\frac{2f_c x}{h_1}} \qquad (4)$$

where σ_{y_0} – stress at the root of a crack, Pa; f_c – coefficient of contact friction; h_1 – sample height, and specific pressure

$$p=\sigma_{y_0}\frac{h_1}{f_c a_1}e^{\frac{2f_c a_1}{h_1}-1}, \qquad (5)$$

where a_1 – sample width.

Determining the maximum specific deformation pressure is sufficient in the field of metal pressure forming. Verification of the approximate method of calculating deformation forces obtained in this way using experimental data confirmed its satisfactory practical accuracy. However, the stresses τ_k are usually not equal to k_n. Moreover, these stresses are not equal to k_n with respect to rock samples. During the deformation of rocks with internal friction, it has a different appearance [13–16].

The M.S. Poliakov Institute of Geotechnical Mechanics of the National Academy of Sciences of Ukraine has developed a theory of local failure, which proves that expression (3) for rock with internal friction has a different form [13–16], which made it possible to analytically determine the stability of rock masses. However, it does not fully take into account the influence of the friction properties of rocks on their stress–strain state and does not reveal the relationship between the derivatives of normal vertical stresses and stresses perpendicular to the compression vector, which affects the distribution of contact stresses.

In the solution of [21], the tangential contact stresses do not depend on the *x*-axis and on the vertical stress variables, but according to the Coulomb-Amonton law, the tangential stresses are directly proportional to the vertical stresses, which we will take into account.

We will supplement the system of differential equilibrium equations (1–2) with an algebraic equation describing the behavior of rocks in compression, which was proposed by [22].

$$\sigma_{x_i}=\frac{2\left(k_n+\mu\sigma_{y_i}\right)}{\cos\rho}\left(\sin\rho-\sqrt{1-b_i^2}\right)+\sigma_y \tag{6}$$

where $b_i=\frac{f_c\left(1-\frac{2y}{h}\right)\sigma_y}{k_n+\mu\sigma_y}$; ρ – angle of internal friction, rad.

In the resulting system of equations, the variables are σ_x, σ_y, τ_{xy}, x, y.

As a result of solving the system of equations, we obtain

$$\frac{\partial\sigma_x}{\partial x}=\left\{1+\frac{2}{\cos\rho}\left[\mu\left(\sin\rho-\sqrt{1-b_i^2}\right)+\frac{b_i f_c k_n\left(1-\frac{2y}{h}\right)}{\sqrt{1-b_i^2}\left(k_n+\mu\sigma_y\right)}\right]\right\}\frac{\partial\sigma_y}{\partial x}, \tag{7}$$

where μ – coefficient of internal friction.

Contact tangential stresses according to the Coulomb-Amonton law

$$\tau_{xy} = \mu\sigma_y \quad . \tag{8}$$

Then we get

$$\frac{\partial\sigma_x}{\partial x} = \frac{-1}{\mu}\left\{1 + \frac{2}{\cos\rho}\left[\mu\left(\sin\rho - \sqrt{1-b_i^2}\right) + \frac{b_i f_c k_n\left(1-\frac{2y}{h}\right)}{\sqrt{1-b_i^2}\left(k_n + \mu\sigma_y\right)}\right]\right\}\frac{\partial\sigma_y}{\partial x} \quad . \tag{9}$$

Let us denote the expression between derivatives by ϖ, we obtain

$$\frac{\partial\sigma_x}{\partial x} = \varpi\frac{\partial\sigma_y}{\partial y} \tag{10}$$

Using the first differential equations (1) and (10), we have ϖ

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} \equiv \varpi\frac{\partial\sigma_y}{\partial y} + \frac{\partial\tau_{xy}}{\partial y} = 0 \tag{11}$$

Tangential stresses τ_{xy} decrease in absolute value with distance from each contact surface according to Saint-Venant's principle and, at $y=0.5$, rotate to zero, as on the axis of symmetry. Therefore, after integration:

$$\sigma_y = C \cdot e^{\frac{2 f_c x}{\varpi h_1}} \quad . \tag{12}$$

Considering that constant C will be equal to σ_{y_0} – the vertical normal stress at the corner point of the sample $x=0$. Then, taking into account the boundary conditions, we have:

$$\sigma_y = \sigma_{y_0} \cdot e^{\frac{2 f_c x}{\varpi h_1}} \tag{13}$$

Using this convention, we can write that the tangential stresses

$$\tau_k = f_c \sigma_{y_0} \cdot e^{\frac{2 f_c x}{\varpi h_1}} \tag{14}$$

3. Results and discussion

Based on the above decision, it was found that the ratio between the derivatives of normal stresses arising in the direction perpendicular to the compression vector to the compression stresses is inversely proportional to the internal friction and directly proportional to the sum of the contact and internal friction. This, in turn, made it possible to refine the distribution of contact normal and tangential stresses [22] against the known distribution under the condition $\frac{\partial \sigma_x}{\partial x} = \frac{\partial \sigma_y}{\partial y}$. For a clearer understanding, let us compare the calculation of the strength limit of a sample for a truncated wedge-shaped fracture using equation (10). Taking into account the destruction of the sample along two cracks, we can write that the distribution of normal stresses on the area that does not come out from under the load during development from the upper corners has the form

$$\sigma_{y_i} = \sigma_{y_0} ¿ e^{\frac{2 f_c (a_1 - 2 x_\xi)}{\varpi h_1}} . \tag{15}$$

where σ_{y_i} – vertical normal stress at the crack tip, Pa; x_ξ – crack development coordinate.

The specific contact force at the given location is determined by

$$p = \frac{2\sigma_{y_0}}{a_1 - 2x_\xi} \left(\int_0^t e^{\frac{2 f_c t}{\omega h}} dt \right) = \sigma_{y_\xi} \frac{\varpi h}{f_c (a_1 - 2x_\xi)} \left(e^{\frac{f_c (a_1 - 2x_\xi)}{\varpi h}} - 1 \right) . \tag{16}$$

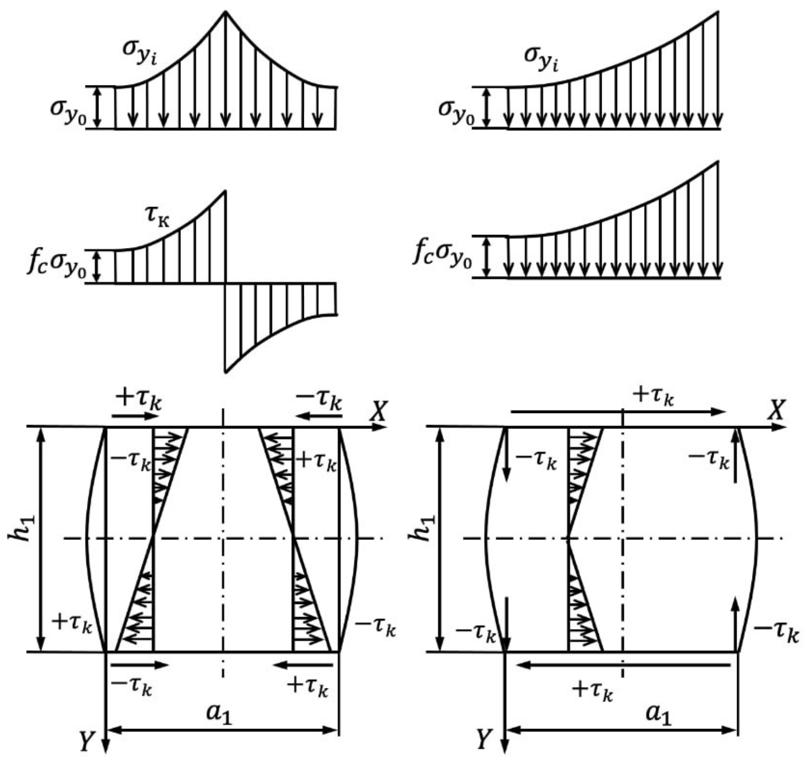
Thus, a refined method for calculating the distribution of normal and tangential stresses has been developed by solving two differential and one algebraic equations of equilibrium of forces from normal and perpendicular stresses in accordance with the requirements of the theory of mechanics of deformed objects.

The tangential stress $\tau_{xy} = \tau_{xy}(x, y)$ on the contact surface is equal to the tangential stress τ_k, which is caused by friction between the tool and the rock under load. The distribution of contact normal and internal tangential stresses under symmetrical and asymmetrical loading is shown in Fig. 1.

By obviousness, we assume that on the horizontal line of symmetry of the sample τ_{xy} is equal to zero. Therefore, assuming that the decay of contact tangential stresses from friction along the longitudinal axis occurs according to a linear law, we write that the internal tangential stresses are determined by the formula

$$\tau_{xy}=\tau_k\left(1-\frac{2y}{h_1}\right)=f_c\sigma_{y_\xi}\left(1-\frac{2y}{h_1}\right)e^{\frac{2f_c x}{h_1}}. \quad (17)$$

where τ_k – internal tangential stresses, Pa;



a – symmetric *b* – asymmetric

Figure 1 – Diagrams of contact normal σ_y and internal tangential τ_k stresses at $\tau_k=f_c\sigma_{y_i}$ by the author, [22]

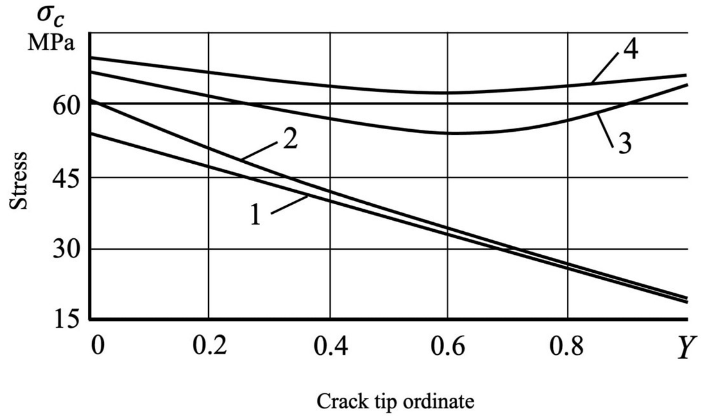
This equation allows determining internal tangential stresses. It is important to note that the equations given contain the geometric dimensions of the objects under study, which allows the results of the calculation of the parameters of the rock sample to be transferred in accordance with the laws of stress distribution to a rock mass of any size, excluding the influence of the scale geometric effect, (but not the influence of layering and fracturing of the rock).

The parameters of the stress–strain diagrams of truncated wedge-shaped samples are determined by the iteration method. For this purpose, the sample is divided into several layers *n*. The solution allows determining the limit values of vertical and perpendicular to them - horizontal normal stresses, deformations and a number of intermediate parameters at the tops of cracks during their development.

Figure 2 shows the calculated diagrams "stress–crack tip ordinate" for comparison, and Figure 3 shows the calculated "stress–deformation" diagrams, the parameters of which are determined by formulas (15) and (16) using the distribution of contact stresses by the improved and traditional methods.

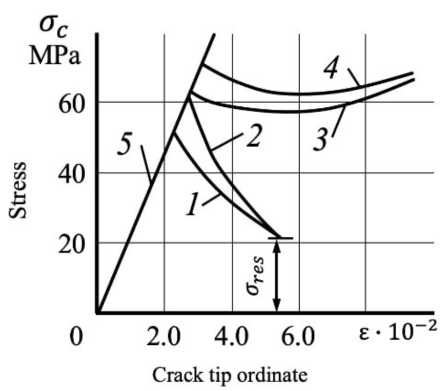
At high values of internal friction angles, the decay modulus takes the form of a concave quadratic function, while the residual strength is not preserved, but the current strength value using the improved method of stress distribution is always lower than that using the traditional method. This conclusion implies that the decay module depends on the physical, mechanical, and friction properties of rocks and is not a constant value.

Figure 3 shows line 5 of the true "stress–longitudinal deformation" diagram, which is determined without taking into account the ratio of the current bearing area of the sample to its initial cross-section. This line is straight, and the trajectories of the maximum effective tangential stresses (TMETS) on the outer branches of the conditional diagrams are shown as curved lines.



1, 2 – $\rho=45°$, $f_c=0.25$; 3, 4 – $\rho=55°$, $f_c=0.25$

Figure 2 – Calculated diagrams of "stress - crack tip ordinate" according to the improved (1, 3) and E.P. Unskov's law (2, 4) distributions for two TMETS ξ at $k_n=10.0\ MPa$



1, 2 – $\rho=45°$, $f_c=0.25$; 3, 4 – $\rho=55°$, $f_c=0.25$; 5 – line of the true diagram

Figure 3 – Calculated conditional diagrams of "stress - crack tip ordinate" according to the improved (1, 3) and E.P. Unskov's law (2, 4) distributions for two TMETS ξ at $k_n = 10.0\ MPa$

The proposed method demonstrates that taking into account perpendicular stresses and contact friction significantly changes the stress distribution compared to classical solutions. The decrease in current strength values observed in the "stress–crack tip coordinate" and "stress–strain" diagrams (Figs. 2–3), especially at low internal friction angles, indicates that traditional approaches may overestimate the stability of rocks under certain loading conditions. This discrepancy becomes more noticeable under conditions of high internal friction, when the dependence graph takes on a concave quadratic shape, which once again confirms the need to refine the model. It should be noted that the correspondence of the residual strength at the points of breakage (for example, $\rho = 45°$, $f_c = 0.25$) is consistent with empirical observations of brittle destruction of prismatic samples, which brings theoretical and practical conclusions closer together.

The linearity of the true stress–strain diagram (Fig. 3, line 5) contrasts with the curved trajectories of traditional diagrams, emphasizing the influence of contact friction and geometric scaling effects. This conclusion is of direct practical importance: eliminating the geometric scale effect. The method allows the results of laboratory studies to be extrapolated to rock masses of arbitrary sizes, which is an important advantage for assessing stability in deep mining or large-scale excavation works. However, the dependence of the deformation modulus on rock properties (e.g., friction coefficients) means that calibration of the input data for a specific rock site remains important.

4. Conclusions

A method has been developed for calculating the parameters of the "stress–crack tip ordinate" and "stress–deformation" diagrams with an improved distribution of normal and tangential contact stresses, which can be used for brittle, relatively homogeneous rocks under static loading.

A comparison of the calculated "stress–crack tip ordinate" and "stress–deformation" diagrams using the improved method and the method of E.P. Unskov shows that the level of current strength values on the outer branches of the diagrams decreases with the development of two cracks at small angles of internal friction, while the decline module according to the improved distribution has a lower value than according to the traditional method, with the exception of points *bb* and *b'b'* where two TMETS exit onto the lower contact surface, in which case, at $\rho = 45°$, $f_c = 0.25$ the residual strength has the same values.

At high values of internal friction angles, the nature of the decay module changes. It takes the form of a concave quadratic function, while the equality of residual strength is not preserved, but the current value of strength according to the improved stress distribution method is always less than that according to the traditional method.

The proposed approach requires further study of the influence of contact friction on the stress–strain state not only of homogeneous but also of layered rocks, taking into account their moisture content and other physical and mechanical factors.

Conflict of interest

Authors state no conflict of interest.

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About the authors

Malich Mykola, Candidate of Sciences in Engineering, Ukrainian State University of Science and Technology, Department of Industrial Engineering, Dnipro, Ukraine, n22051957m@gmail.com ***(Corresponding author)*** ORCID ***0000-0003-4967-5378***

Katan Volodymyr, Candidate of Sciences in Physics and Mathematics, Oles Honchar Dnipro National University, Dnipro, Ukraine, katanvladimir@gmail.com, ORCID ***0000-0001-9167-2619***

Laikov Dmytro, Postgraduate student, Ukrainian State University of Science and Technology, Department of Industrial Engineering, Dnipro, Ukraine, luckydnepr@gmail.com, ORCID ***0009-0003-6852-2825***

Kravchenko Kostiantin**,** Candidate of Sciences in Engineering, Dnipro University of Technology, Dnipro, Ukraine, kravchenko.k.v@nmu.one, ORCID ***0000-0002-7598-8979***

ДО ДІАГРАМ РУЙНУВАННЯ ПРИЗМАТИЧНИХ ЗРАЗКІВ З ПОКРАЩЕНИМ ЗАКОНОМ РОЗПОДІЛУ КОНТАКТНИХ НАПРУЖЕНЬ

Маліч М., Катан В., Лайков Д., Кравченко К.

Анотація. Видобуток і подальша переробка корисних копалин пов'язані з удосконаленням існуючих та розробкою нових ресурсозберігаючих технологічних рішень з шахтним та відкритим циклом робіт. Найбільш витратною технологією переробки мінеральної сировини є власне дезінтеграція, на яку витрачається близько 20% загального виробництва електроенергії та до 50% загальних капітальних і експлуатаційних витрат. Для визначення стійкості породного масиву у позамежному стані використовують діаграму «напруження - поздовжня деформація» гірських порід. Для аналітичного визначення цих діаграм потрібно знати розподіл контактних нормальних і дотичних напружень. За класичними рішеннями цієї задачі розподіл напружень визначали за методами Л. Прандтля та Є.П. Унксова. Однак ці методи не враховували виникнення напружень, які перпендикулярні вектору стискання.

В статті наведено подальший розвиток методу уточненого визначення розподілу контактних напружень та побудови діаграми «нормальне напруження - поздовжня деформація» і граничних кривих з урахуванням контактного тертя при навантаженні. Проведено порівняльну оцінку запропонованого методу з діаграмами, побудованими за класичним методом. Запропонований метод дозволяє визначати межу міцності та залишкову міцність зразків гірських порід за параметрами, які простими методами можуть бути встановлені експериментально в лабораторіях гірничодобувних підприємств. Результати можуть бути використані для контролю стану гірничого масиву та ефективного руйнування при дезінтеграції гірської породи.

Таким чином запропоновано вдосконалений метод визначення розподілу контактних нормальних та дотичних напружень у призматичних зразках крихких, відносно однорідних гірських порід при стисканні, який враховує напруження, перпендикулярні до вектора навантаження та дозволяє точніше оцінити напружено-деформований стан зразків із урахуванням контактного. Встановлено, що при особливо при малих кутах внутрішнього тертя, рівень поточних напружень за запропонованим методом є нижчим порівняно з класичними рішеннями.

Розроблений підхід дозволяє уникнути впливу масштабного ефекту і переносити результати лабораторних досліджень на реальні неушкоджені та відносно однорідні масиви. Результати мають практичне значення для оцінки стійкості гірських масивів та підвищення ефективності руйнування порід під час видобутку та дезінтеграції твердих корисних копалин.

Ключові слова: контакт, тертя, діаграма, порода, деформація, напруження, руйнування